

π NN Coupling Constants from NN Elastic Data between 210 and 800 Mev

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High partial waves for pp and np elastic scattering are examined critically from 210 to 800 MeV. Non-OPE contributions are compared with predictions from theory. There are some discrepancies, but sufficient agreement that values of the πNN coupling constants g_0^2 for π^0 exchange and g_c^2 for charged π exchange can be derived. Results are $g_0^2 = 13.91 \pm 0.13 \pm 0.07$ and $g_c^2 = 13.69 \pm 0.15 \pm 0.24$, where the first error is statistical and the second is an estimate of the likely systematic error, arising mostly from uncertainties in the normalisation of total cross sections and $d\sigma/d\Omega$.

1 Introduction

There has recently been controversy over the magnitude of the πNN coupling constant. Prior to this controversy, the accepted value for many years was that determined by Bugg, Carter and Carter [1]: $f^2 = 0.0790(10)$, where the error is given in parentheses. Defining

$$g^2 = f^2(2M/\mu_c)^2, \quad (1)$$

where M is the mass of the proton and μ_c the mass of the charged pion, this value of f^2 corresponds to $g^2 = 14.28(18)$. This determination, based on the fixed t dispersion relations for the $B^{(+)}$ amplitude in πN elastic scattering, refers to exchange of charged pions at the nucleon pole: $\pi^- p \rightarrow n$. Since 1987, the Nijmegen group has challenged this value in a series of papers [2-4], analysing NN elastic data up to 350 MeV with a potential model. They find equal coupling constants for exchange of neutral and charged pions within experimental error: $f_0^2 = 0.0749(7)$ or $g_0^2 = 13.54(13)$, $f_c^2 = 0.0741(5)$ or $g_c^2 = 13.39(9)$, and a global average $f^2 = 0.0749(4)$ or $g^2 = 13.54(7)$.

In 1990, Arndt et al. [5] obtained the value $f^2 = 0.0735(15)$, $g^2 = 13.28(27)$ from an analysis of new πN data. It transpires that they discarded much of the best but older data; a re-analysis, using all available data, and with careful attention to Coulomb effects, gives the result [6]: $f^2 = 0.0771(14)$, $g^2 = 13.94(25)$. Meanwhile Machleidt and Sammarruca [7] have tested these results against precise information from the deuteron quadrupole moment and asymptotic D/S state ratio. They come down marginally in support of the higher values of f^2 and g^2 . At the recent Boulder conference, Arndt et al. [8] revised their determination upwards to $g^2 = 13.72(15)$, but stated that further analysis is still in progress, so this value should be considered preliminary.

In this conflicting situation, it is relevant to examine NN elastic scattering data above the energy range considered by the Nijmegen group. These data have the virtue of being very precise and are a potential source of information on g_0^2 , the coupling constant for π^0 exchange. However, they do suffer from some difficulties. A minor difficulty is inelasticity above 300 MeV, which introduces more free parameters into the phase shift analysis. However, the serious issue is the separation of OPE from exchange of 2π , 3π , ρ , ω , etc. These will be described here collectively as “heavy boson exchange”

(HBE). In this paper, the conclusion is reached that NN data from 140 to 800 MeV do indeed provide a useful determinations of g^2 .

Pion exchange contributes through the δ amplitude [9] of the form [10] $(\sigma_1 \cdot \mathbf{q})(\sigma_2 \cdot \mathbf{q})/(t - \mu^2)$, where \mathbf{q} is transverse momentum and $t = -q^2$. It may be isolated using the spin dependence of NN elastic scattering. It turns out that this particular amplitude is well measured by Wolfenstein parameters D_{NN} for elastic scattering [11] and by K_{NN} and K_{LL} for np charge exchange. The basic idea is illustrated in Fig. 1. At 0° , amplitudes $\beta \propto (\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n})$ and $\delta \propto (\sigma_1 \cdot \mathbf{q})(\sigma_2 \cdot \mathbf{q})$ are equal, since there is nothing to distinguish the normal \mathbf{n} to the scattering plane and \mathbf{q} in the plane of scattering. The β amplitude varies slowly with q^2 and can be extrapolated securely to $q = 0$. The δ amplitude varies rapidly with q and crosses zero at about $t = -\mu^2$. The difference between them determines the OPE amplitude. Figs. 1(c) and (d) show that the parameters D_{NN} and K_{NN} have striking dips and peaks respectively due to these zeros; these strong features establish the magnitude of the OPE contribution. A detail is that the Coulomb amplitude dominates pp scattering below $q \simeq 40$ MeV/c, so for pp the vital q range is approximately 50 to 250 MeV/c.

A direct analysis of data on Wolfenstein parameters turns out to be difficult, because of interferences and because it is necessary to describe the slowly varying components in a way consistent with the HBE contributions, which are partly determined by other data. We therefore use the full weight of partial wave analysis and the full data base. The OPE contribution is then derived from the tensor components in high partial waves. These are of two sorts: (a) mixing parameters $\bar{\epsilon}$, which mix triplet states having $J = L \pm 1$, and (b) tensor combinations of H and I waves [12]:

$$H_T \propto -6^3 H_4 + 11^3 H_5 - 5^3 H_6 \quad (2)$$

$$I_T \propto -7^3 I_5 + 13^3 I_6 - 6^3 I_7. \quad (3)$$

In whatever way the analysis is done, the essential difficulty is to separate the exchange of heavier mesons from OPE. Actually σ and ω exchange contribute only to central and spin-orbit combinations, which are orthogonal to tensor components. This leaves ρ and 3π exchanges, which contribute a slowly varying component to tensor combinations. Because of their large masses, they contribute mostly to low partial waves, notably 3P_0 , 3P_1 and 3P_2 [13], where they may be determined phenomenologically. It is well known that ρ exchange cuts off the π tensor amplitude below $r \simeq 1$ fm.

In this paper, we will use the HBE contributions as determined by the “Bonn peripheral model”. This is an extension of the model for higher partial waves developed in section 5 of Ref. [14] above pion-production threshold. It uses the approach described in Appendix B (Model II) of Ref. [13]. The ρ coupling is taken from the work of Höhler and collaborators [15] and the correlated 2π S-wave contribution from Durso et al. Only the ω coupling is adjusted so as to fit 3F_4 and 1G_4 . The model also includes $\Delta(1232)$ -isobars in intermediate states which are excited via π and ρ exchange and provide inelasticity as well as additional intermediate-range attraction.

The program of this work is to examine critically each of the high partial waves, so as to form an opinion of what is (or is not) understood, before passing judgement on g^2 . The predictions by the peripheral Bonn model are in excellent agreement with pp data for partial waves with $J \geq 4$, with the exception of 3H_4 . Corrections can be made for this small discrepancy, leaving what appears to be a reliable determination of neutral π exchange and the corresponding coupling constant g_0^2 .

For np charge exchange, OPE is three times larger than for pp elastic scattering, so one might hope to derive precise values of the charged coupling constant g_c^2 . The experimental data on Wolfenstein parameters are as good for K_{NN} in charge exchange as for D_{NN} in pp elastic scattering. Unfortunately, theory is not in such good shape, and there are clear discrepancies between HBE predictions and experiment for $I = 0$ G and H waves. We find that it is still possible to make a reasonably accurate determination of g_c^2 from $\overline{\epsilon}_5$ and higher partial waves.

Section 2 analyses pp scattering and arrives at a determination of g_0^2 . Section 3 analyses np data and g_c^2 . Section 4 comments on systematic errors and other determinations of g^2 . Section 5 presents conclusions.

2 High partial waves for pp scattering

There have been extensive and very accurate measurements of Wolfenstein parameters at TRIUMF from 210 to 515 MeV, at PSI by the Geneva group up to 580 MeV and at LAMPF from 485 to 800 MeV. Where they overlap, these experiments agree well, so one can have confidence in the data. In fact, what is needed to determine g^2 is a product of Wolfenstein parameters and $d\sigma/d\Omega$, and the bigger problems reside in the latter, where absolute

normalisation presents experimental difficulties. Here the optical theorem is some help, by relating the imaginary part of the spin averaged amplitude to the total cross section via the optical theorem. A precise determination of g^2 depends on high absolute accuracy in all these data. Results will be compared at eight energies from 210 to 800 MeV. Because data come from a variety of experiments using different techniques, one gets some idea of systematic errors. At the Gatchina energy of 970 MeV, there are no accurate measurements at the small angles required for present purposes. Likewise, around 142 MeV, there are no measurements of Wolfenstein parameters below 30°. So these two energies are omitted.

Table 1 compares predictions for mixing parameters $\bar{\epsilon}_4$ and $\bar{\epsilon}_6$ with phase shift analysis. For $\bar{\epsilon}_4$, agreement is remarkably good right up to 800 MeV. At the upper energies, HBE contributions are becoming uncomfortably large. However, in view of the agreement for $\bar{\epsilon}_4$, one can have great confidence in $\bar{\epsilon}_6$, where HBE contributions are much smaller. Free fits to $\bar{\epsilon}_6$ at 580 and 800 MeV give satisfactory agreement with predictions.

The agreement for $\bar{\epsilon}_4$ is so good that one might wonder whether this parameter has already been used in optimising theoretical predictions. This is not so. We stress again that except for the ω (which does not create any tensor force and thus does not contribute directly to $\bar{\epsilon}_J$) all parameters in the theoretical model are taken from independent sources and have not been fitted to data. So the agreement of $\bar{\epsilon}_4$ with prediction up to 800 MeV is a real success and not a circular argument.

Table 2 makes similar comparisons for 1G_4 and H waves. For 1G_4 , agreement is excellent, but the HBE contribution is large compared with OPE.

For 3H_4 , there is a definite discrepancy beginning at 325 MeV. This raises the question of how far to trust HBE predictions for 3K_6 . In a classical approximation, angular momentum L is related to impact parameter r and momentum k by $\sqrt{L(L+1)} = kr$. This suggests that agreement for 3H_4 up to 210 MeV implies agreement for 3K_6 up to a lab energy $T = 210 \times (6 \times 7) / (4 \times 5) \simeq 450$ MeV. Calculations of HBE support this rough classical notion for high partial waves. Using this prescription, the discrepancies for 3H_4 have been used to estimate small corrections for 3K_6 from 515 to 800 MeV, Table 3. In practice, it turns out that this refinement has an effect on g^2 rather below statistical errors.

For 3H_5 and 3H_6 , agreement is satisfactory up to 580 MeV; above this

energy, small systematic discrepancies begin to appear. It implies that 3K_7 and 3K_8 should be reliable up to 800 MeV and this appears to be so experimentally within two standard deviations.

Table 4 summarises the measure of agreement in the various partial waves. A tick indicates agreement, a cross disagreement, L indicates that the HBE contribution is too large for comfort (a subjective judgement) and C indicates that a correction has been applied.

Table 5 gives values of g_0^2 , depending on a variety of assumptions. In the first column, $\bar{\epsilon}_4$ and 3H_5 are used, together with all parameters from $\bar{\epsilon}_6$ upwards. (At 210 MeV, 3H_4 is also used). In Column 2, $\bar{\epsilon}_4$ is used but H waves are left free. In Column 3, only $\bar{\epsilon}_6$ and higher waves are used; this is very conservative, almost certainly too conservative at 210 and 325 MeV.

There is satisfactory consistency over most of Table 5. In trying to derive a mean value for g_0^2 , it is necessary to steer a middle course between (a) using only the very high partial waves, hence incurring a large error, or (b) risking that errors in HBE affect g_0^2 . We have chosen to use HBE contributions up to the energies where they become 20 - 25% of OPE. The reasoning is as follows. We shall find errors on g^2 of about $\pm 2\%$. It seems reasonable to believe HBE contributions to 10% of their magnitudes in view of the excellent agreement for $\bar{\epsilon}_4$ (and later $\bar{\epsilon}_3$ and $\bar{\epsilon}_5$). This means that we use $\bar{\epsilon}_4$, 3H_5 , and 1I_6 up to 515 MeV. We cut off 3H_4 above 210 MeV, because of the systematic discrepancies in Table 2. It implies taking g_0^2 from column 1 of Table 5 up to 515 MeV and column 3 thereafter. The result is a weighted mean

$$g_0^2 = 13.91 \pm 0.13. \quad (4)$$

The reader can easily make any other particular choice.

At 720 MeV, the data base is decidedly thin, and freeing $\bar{\epsilon}_4$ leads to considerable latitude in the phase shifts. The value $g_0^2 = 10.81 \pm 0.83$ at this energy in the third column of Table 5 is four standard deviations from the mean and has therefore been discarded.

As stated in the Introduction, the OPE amplitude is being determined essentially between $q = 50$ and 250 MeV/c, i.e. at a mean value of $t \simeq -\mu^2$. One has to worry about the effect of a form factor. We write the OPE amplitude proportional to

$$\frac{g^2}{t - \mu^2} \frac{\mu^2 - \Lambda^2}{t - \Lambda^2} = g^2 \left(\frac{1}{t - \mu^2} - \frac{1}{t - \Lambda^2} \right). \quad (5)$$

The Bonn fit to lower partial-wave phase parameters requires $\Lambda = 1.3$ to 1.7 GeV/c^2 [13]. It is then straightforward to make a partial wave decomposition of the term $g^2/(t - \Lambda^2)$. The result, using $\Lambda = 1.4 \text{ GeV}/c^2$, is a perturbation to $\bar{\epsilon}_4$ of $+0.04^\circ$ at 800 MeV and less for lower energies and higher partial waves. This is completely negligible. Physically it corresponds to the fact that the distant pole at $\Lambda = 1.4 \text{ GeV}/c^2$ affects only low partial waves. We remark, however, that the perturbation for $\bar{\epsilon}_3$ is -0.38° at 800 MeV with $\Lambda = 1.4 \text{ GeV}/c^2$. In the next section, we shall find excellent agreement between $\bar{\epsilon}_3$ and experiment up to 650 MeV . This is an independent check that Λ cannot be substantially less than $1.4 \text{ GeV}/c^2$, since the correction varies roughly as $1/\Lambda^2$.

We address the question of systematic errors on g_0^2 in Section 4.

3 np Data and g_c^2

Table 6 compares predictions for $\bar{\epsilon}_3$ with experiment. Agreement is satisfactory, except at 800 MeV , where prediction is dropping away from experiment. However, contributions from HBE are too large to allow direct use of $\bar{\epsilon}_3$ in determining g_0^2 . Nonetheless, one can have great confidence in using $\bar{\epsilon}_5$, which is also well determined experimentally. Table 7 shows values of $\bar{\epsilon}_5$ from the phase shift analysis of Bugg and Bryan [16].

For G and H waves, the story is not so nice, Table 8. Above 325 MeV , there are large discrepancies between experiment and Machleidt's predictions for 3G_3 and 3G_4 and a hint of disagreement for 3G_5 . The former discrepancies are certainly real. They are visible in TRIUMF K_{SS} , K_{LS} and A_{NN} data at 425 and 515 MeV and in independent LAMPF data for the same parameters at higher energies. An experimental cross-check is that there is no apparent problem with K_{NN} data, which were measured with the same technique and at the same time as K_{SS} and K_{LS} . Where they overlap near 500 MeV , TRIUMF and LAMPF data agree for A_{NN} and K_{NN} . We note that the fit to 3G_4 at higher energies could be improved by a drastic reduction of g_c^2 (to 11.41 ± 0.19 from 800 MeV data). However, this would result in terrible predictions for $\bar{\epsilon}_3$ and $\bar{\epsilon}_5$. Furthermore, 3G_4 at lower energies ($T \leq 325 \text{ MeV}$) would deteriorate substantially. Thus there is no choice for g_c^2 that would consistently improve 3G_4 at all energies. So the conclusion is that the disagreement must be taken seriously.

At 800 MeV, 1H_5 comes out significantly negative of OPE; 1F_3 shows the same feature from 325 to 800 MeV [16]. Until the problem with the G waves and 1H_5 is understood, one cannot have complete confidence in deriving g_c^2 from np data.

Table 9 summarises the measure of agreement between predictions and $I = 0$ partial waves. From the impact parameter prescription [12], one can estimate perturbations to be applied to I and K waves for the discrepancies in Table 8. The corrections, given in Table 3, are small and have the effect of lowering g_c^2 slightly above 515 MeV. At 800 MeV, the experimental determination of 3I_6 is compatible with the estimated correction. At that energy, experiment is capable of determining partial waves up to 3K_8 .

The result of this analysis is a determination of g_c^2 shown in Table 10 using $\bar{\epsilon}_5$ and higher partial waves. The weighted mean value of g_c^2 is

$$g_c^2 = 13.67 \pm 0.29, \quad (6)$$

where the error has been inflated from the statistical value ± 0.154 in order to cover fluctuations about the mean. Corrections for the pion form factor are again completely negligible.

In view of the excellent agreement between $\bar{\epsilon}_3$ and prediction, an alternative procedure is to use only $\bar{\epsilon}_5$, which is extremely well determined by K_{NN} data and also by K_{LL} from 485 to 800 MeV. We mention in particular the data of Abegg et al. [17] which lead to a very tight constraint on $\bar{\epsilon}_5$. Using this parameter alone leads to

$$g_c^2 = 13.69 \pm 0.15. \quad (7)$$

This value has the virtue of being independent of the small corrections applied to 1H_5 , 3I_5 and 3I_6 . It has a smaller error than eqn. (6) because the fluctuations in ϵ_5 are smaller than those in higher partial waves. It is our preferred value. However, the error in eqn. (7) is purely statistical and we shall show in the next section that systematic errors are likely to be larger.

4 Systematic errors

The errors discussed so far are statistical. We now estimate systematic errors arising from $d\sigma/d\Omega$ and Wolfenstein parameters, which we know to be

decisive in determining g^2 . For pp , it turns out that they are about ± 0.07 for g_0^2 , i.e. slightly less than statistical errors. For g_c^2 the situation is the reverse. Because of systematic uncertainties in normalisation, there is a systematic uncertainty which we tentatively estimate as ± 0.24 .

For pp scattering, the OPE amplitude is determined mostly by $D_{NN} d\sigma/d\Omega$, where in conventional notation [9]

$$D_{NN} = \frac{|\alpha|^2 + |\beta|^2 - |\delta|^2 - |\epsilon|^2 + 2|\gamma|^2}{|\alpha|^2 + |\beta|^2 + |\delta|^2 + |\epsilon|^2 + |\gamma|^2}. \quad (8)$$

The measurement of D_{NN} is made using a polarimeter which is calibrated directly using the polarised beam. At the very small momentum transfers relevant to OPE, any normalisation errors cancel, so this source of systematic errors can probably be neglected. At the most pessimistic, it should be $\leq \pm 1\%$.

The larger problem lies in $d\sigma/d\Omega$. Here we estimate a systematic error of $\pm 1\%$, for reasons we shall now detail. From 500 to 800 MeV, recent measurements of Simon et al. [18] have statistical errors of $\pm 0.5\%$ and an absolute accuracy of about $\pm 0.5\%$. From 500 to 580 MeV, there are similarly precise data of Chatelain et al. [19], and from 300-500 MeV precise 90° data of Ottewell et al. [20] with normalisation errors of $\pm 1.8\%$. There are many measurements of the shape of $d\sigma/d\Omega$, but with larger normalisation errors. These can all be tied together with the pp total cross section data of Schwaller et al. [21] from 179 to 555 MeV, having systematic errors of $\pm 0.8\%$ and statistics of $\pm 1\%$. One finds in phase shift analysis that there are no particular conflicts amongst all these data, so we conclude with an overall impression of a $\pm 1\%$ error in $d\sigma/d\Omega$.

As regards OPE, the formula for D_{NN} depends essentially on $|\beta|^2 - |\delta|^2$, so a 1% systematic error translates into a $\pm 0.5\%$ error in the scale of the OPE amplitude, i.e. an uncertainty in g_0^2 of $\simeq \pm 0.07$. This estimate has been checked by dropping in turn various sets of data from the phase shift analysis and deliberately altering normalisations.

For np charge exchange, the question of absolute normalisation has been reviewed recently by McNaughton et al. [22]. For Wolfenstein parameters, the normalisation error is estimated at $\pm 1.8\%$. However, the situation for $d\sigma/d\Omega$ is less satisfactory. There are few measurements aiming at good absolute normalisation. The difficulty lies in knowing the absolute flux of the neutron beam. Keeler et al. [23] took great pains over this and claim an

absolute accuracy of $\pm 1.6\%$. However, even they admit to uncertainties of $\pm 3\%$ for the relative normalisations of forward scattering (where the neutron is detected) and charge exchange (where the proton is detected). Carlini et al. [24] present data at 800 MeV with good absolute normalisation in the forward hemisphere.

The situation is confused by substantial discrepancies over np total cross sections. These disagree amongst themselves and also disagree with the differential cross sections we have just described. Lisowski et al. [25] present data from 40 to 770 MeV with very high statistics; they claim absolute normalisation of $\leq \pm 1\%$. Unfortunately their data differ by 6% from most other data. These other data, although of lower statistical accuracy, were mostly measured with monoenergetic neutron beams and liquid hydrogen targets, whereas Lisowski et al. worked with $CH_2 - C$ difference and a continuous neutron spectrum, a less attractive technique as regards absolute normalisation. Our phase shift solutions settle midway between the Lisowski et al. results and the rest, but can be driven to fit either without too great a penalty in χ^2 . Keeler et al. $d\sigma/d\Omega$ data definitely show a preference against the Lisowski et al. data. If the latter are dropped, g_c^2 rises systematically at all energies and averages to 13.90. It therefore seems essential to allow $\pm 3\%$ normalisation uncertainty for $d\sigma/d\Omega$. Adding in quadrature the $\pm 1.8\%$ uncertainty in K_{NN} , we arrive at $\pm 3.5\%$ normalisation error in $|\beta|^2 - |\delta|^2$, i.e. $\pm 1.75\%$ systematic uncertainty in the OPE amplitude. This translates to a systematic error in g_c^2 of ± 0.24 .

We now comment briefly on other determinations of g^2 . The Nijmegen group has reported extensively on determination of g^2 from data up to 350 MeV. There are two comments to be made on this work. Firstly, it must contain similar uncertainties about HBE buried in the short-range potentials, but different in detail to those appearing here. Secondly, it is interesting that they assert strongly that their values refer to the pion pole at $t = \mu^2$. Some years ago, forward dispersion relations were used [26] to find g_0^2 from the u channel pole below threshold. This work was repeated with later data by Grein and Kroll [27]. It seems likely that the Nijmegen work is likewise finding g_0^2 from the u channel contribution. Their potential model calculation, via the Schrödinger equation, will include rescattering which appears in dispersion integrals. The pion pole is only 10 MeV below threshold and makes its dominant contribution at low energies, through the energy dependence of 1S_0 and 3S_1 . The Nijmegen group indeed finds that their

greatest sensitivity is to data in the 10-30 MeV range. They observe no particular sensitivity to Wolfenstein parameters.

If this assessment of the origin of their determination of g_0^2 is correct, there is a corollary. One needs to be cautious about Coulomb corrections, which are large at low energies and have a dramatic effect on the scattering length. The Nijmegen group has been fastidious about the long range component of the Coulomb potential. However, one needs to ask whether short-range Coulomb corrections to the scattering length and effective range are adequately understood (for example those due to mass differences between u and d quarks).

The second route for determining g_c^2 is from πN fixed t dispersion relations. This method should be more reliable for g_c^2 than using NN data. Firstly, it evaluates the pole value directly. The nucleon pole lies between the s and u channel regions, and is obtained by interpolation rather than from an extrapolation. Secondly, the relevant $B^{(+)}$ amplitude is determined primarily by P_{33} , indeed by the width of the P_{33} resonance. This partial wave is determined easily and accurately by total cross section measurements up to 300 MeV, aided by $d\sigma/d\Omega$ and A_{0N} which determine small partial waves [28]. Total cross section measurements are a well tested technique, not only for πN scattering but for many other reactions; worldwide agreement is characteristically at the 1 – 2% level between different groups. Thirdly, Coulomb corrections cancel to first order in the $B^{(+)}$ amplitude, which is a symmetric combination of $\pi^+ p$ and $\pi^- p$. Nonetheless, caution is essential because the P_{33} amplitude has a significant splitting of mass and width between Δ^{++} and Δ^0 states and Coulomb barrier corrections are sizeable. The charge dependence is accurately measured experimentally and the Coulomb corrections have been studied in great depth theoretically [29]. So it appears that Coulomb corrections in πN can be handled accurately.

5 Concluding remarks

NN data give consistent determinations of both g_0^2 and g_c^2 using data from 140 to 800 MeV. The essential source of the information lies in precise measurements of Wolfenstein parameters D_{NN} , K_{NN} and K_{LL} , together with $d\sigma/d\Omega$. Our preferred values are

$$g_0^2 = 13.91 \pm 0.13 \pm 0.07,$$

$$g_c^2 = 13.69 \pm 0.15 \pm 0.24.$$

The latter value increases to 13.90 if the total cross section data of Lisowski et al. are dropped. The choice of determinations in Tables 5 and 10 is somewhat subjective, but the interested reader can easily make his or her own choice. Results are consistent with absence of charge dependence.

The result for g_0^2 is significantly larger than that of the Nijmegen group, $g_0^2 = 13.54 \pm 0.13$. It is close to the latest value g_c^2 from ref. [6] and marginally above Arndt's latest value $g_c^2 = 13.72 \pm 0.15$. Further precise measurements of Wolfenstein parameters in the 140-300 MeV range, below the inelastic threshold, should allow even further improvement in accuracy for g_0^2 , and such measurements are in progress at IUCF [30]. At these energies, HBE corrections to $\bar{\epsilon}_4$, G and H waves are surely small and accurately determined from a global fit to low partial waves and higher energy data. Errors of ± 0.005 on both Wolfenstein parameters and $d\sigma/d\Omega$ may be achievable and would determine g_0^2 with an accuracy of about ± 0.06 . For np measurements, prospects of further improvements are not good, because of the great difficulty of measuring $d\sigma/d\Omega$ absolutely in charge exchange.

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Figure Caption

Fig. 1. Real parts of β and δ amplitudes for (a) pp elastic scattering (b) np charge exchange at 515 MeV, (c) and (d) corresponding values of D_{NN} and K_{NN} .

T(MeV)	OPE	HBE	OPE+HBE	Experiment	Discrepancy	Parameter
210	-1.128	0.021	-1.107	-1.29 ± 0.09	-0.18 ± 0.09	$\bar{\epsilon}_4$
325	-1.614	0.112	-1.502	-1.47 ± 0.07	0.03 ± 0.07	
425	-1.939	0.262	-1.677	-1.66 ± 0.04	0.01 ± 0.04	
515	-2.179	0.455	-1.724	-1.74 ± 0.07	-0.02 ± 0.07	
580	-2.329	0.625	-1.704	-1.67 ± 0.06	0.03 ± 0.06	
650	-2.473	0.827	-1.646	-1.44 ± 0.06	0.21 ± 0.06	
720	-2.601	1.037	-1.564	-1.66 ± 0.04	-0.09 ± 0.04	
800	-2.732	1.276	-1.456	-1.42 ± 0.04	0.04 ± 0.04	
210	-0.332	0.000	-0.332			$\bar{\epsilon}_6$
325	-0.554	0.003	-0.551			
425	-0.717	0.010	-0.707			
515	-0.843	0.020	-0.823			
580	-0.918	0.031	-0.887	-0.99 ± 0.05	-0.10 ± 0.05	
650	-1.003	0.046	-0.957			
720	-1.075	0.063	-1.022			
800	-1.149	0.087	-1.062	-0.98 ± 0.04	0.08 ± 0.04	

Table 1: Comparing predictions for $\bar{\epsilon}_4$ and $\bar{\epsilon}_6$ with experiment. Units are degrees.

T(MeV)	OPE	HBE	OPE+HBE	Experiment	Discrepancy	Parameter
210	0.700	0.280	0.980	1.07 ± 0.07	0.09 ± 0.07	1G_4
325	0.905	0.761	1.666	1.68 ± 0.05	0.01 ± 0.05	
425	1.011	1.345	2.356	2.31 ± 0.05	-0.04 ± 0.05	
515	1.074	1.990	3.064	3.02 ± 0.04	-0.04 ± 0.04	
580	1.106	2.495	3.601	3.68 ± 0.06	0.08 ± 0.06	
650	1.132	2.964	4.098	4.26 ± 0.06	0.16 ± 0.06	
720	1.150	3.246	4.396	4.15 ± 0.09	-0.25 ± 0.09	
800	1.164	3.332	4.496	4.79 ± 0.07	0.29 ± 0.07	
580	0.542	0.292	0.834	0.87 ± 0.04	0.03 ± 0.04	1I_6
800	0.620	0.531	1.151	1.24 ± 0.05	0.09 ± 0.05	
210	0.309	0.027	0.336	0.31 ± 0.05	-0.03 ± 0.05	3H_4
325	0.539	0.061	0.600	0.49 ± 0.05	-0.11 ± 0.05	
425	0.723	0.073	0.796	0.50 ± 0.05	-0.29 ± 0.05	
515	0.875	0.052	0.927	0.37 ± 0.05	-0.56 ± 0.05	
580	0.977	0.006	0.983	0.51 ± 0.04	-0.47 ± 0.04	
650	1.079	-0.083	0.996	0.59 ± 0.06	-0.41 ± 0.06	
720	1.176	-0.231	0.945	0.48 ± 0.06	-0.46 ± 0.06	
800	1.279	-0.481	0.798	0.49 ± 0.04	-0.31 ± 0.04	
580	0.314	0.049	0.363	0.40 ± 0.04	0.03 ± 0.04	3K_6
800	0.444	0.055	0.499	0.42 ± 0.04	-0.08 ± 0.04	
325	-1.286	0.183	-1.103	-1.19 ± 0.06	-0.09 ± 0.06	3H_5
425	-1.645	0.333	-1.312	-1.36 ± 0.05	-0.05 ± 0.05	
515	-1.926	0.501	-1.425	-1.56 ± 0.06	-0.13 ± 0.06	
580	-2.109	0.635	-1.474	-1.58 ± 0.07	-0.11 ± 0.07	
650	-2.289	0.772	-1.517	-1.66 ± 0.08	-0.15 ± 0.08	
720	-2.454	0.870	-1.584	-1.28 ± 0.07	0.30 ± 0.07	
800	-2.626	0.917	-1.709	-1.48 ± 0.07	0.22 ± 0.07	
580	-0.847	0.103	-0.744	-0.75 ± 0.07	-0.01 ± 0.07	3K_7
800	-1.124	0.194	-0.930	-0.96 ± 0.07	-0.03 ± 0.07	
210	0.121	0.063	0.184	0.20 ± 0.04	0.02 ± 0.04	3H_6
325	0.230	0.185	0.415	0.42 ± 0.04	0.00 ± 0.04	
425	0.323	0.331	0.654	0.83 ± 0.04	0.18 ± 0.04	
515	0.402	0.475	0.877	0.81 ± 0.02	-0.07 ± 0.02	
580	0.456	0.587	1.043	1.05 ± 0.03	0.01 ± 0.03	
650	0.512	0.707	1.219	1.37 ± 0.05	0.15 ± 0.05	
720	0.564	0.823	1.387	1.64 ± 0.05	0.25 ± 0.05	
800	0.621	0.943	1.56417	1.79 ± 0.02	0.23 ± 0.02	
580	0.167	0.105	0.272	0.40 ± 0.06	0.13 ± 0.06	3K_8
800	0.246	0.207	0.453	0.50 ± 0.02	0.04 ± 0.02	

Table 2: Comparison of G , H , I and K waves (degrees) with experiment.

T(MeV)	3K_6	1H_5	3I_5	3I_6
210	0.004	0.004	-0.030	0.032
325	0.013	-0.021	-0.083	0.088
425	0.026	-0.086	-0.130	0.149
515	0.018	-0.182	-0.130	0.150
580	0.007	-0.283	-0.105	0.121
650	-0.014	-0.421	-0.071	0.083
720	-0.026	-0.590	-0.036	0.041
800	-0.056	-0.821	0.000	0.000

Table 3: Corrected HBE values from an impact parameter prescription. Units are degrees.

T(MeV)	$\bar{\epsilon}_4$	3H_4	1G_4	3H_5	3H_6	$\bar{\epsilon}_6$	3K_6	1I_6	3K_7	3K_8
210	✓	✓	L	✓	L	✓	✓	✓	✓	✓
325	✓	✓	L	✓	L	✓	✓	✓	✓	✓
425	✓	✗	L	✓	L	✓	✓	✓	✓	✓
515	✓	✗	L	✓	L	✓	C	✓	✓	✓
580	L	✗	L	L	L	✓	C	L	✓	✓
650	L	✗	L	L	L	✓	C	L	✓	✓
720	L	✗	L	L	L	✓	C	L	✓	✓
800	L	✗	L	L	L	✓	C	L	✓	✓

Table 4: Summary of the measure of agreement between HBE and experiment for pp ; ✓ indicates agreement, ✗ disagreement, L indicates that HBE is uncomfortably large and C indicates that an empirical correction has been applied using data on 3H_4 .

T(MeV)	Using $\bar{\epsilon}_4$ and 3H_5	Using $\bar{\epsilon}_4$	$\bar{\epsilon}_6$ upwards
210	15.01 ± 0.46	15.79 ± 0.48	15.13 ± 0.75
325	14.23 ± 0.33	13.99 ± 0.35	14.70 ± 0.57
425	13.95 ± 0.26	13.41 ± 0.34	13.44 ± 0.38
515	14.02 ± 0.21	14.61 ± 0.25	14.58 ± 0.34
580	13.75 ± 0.35	13.74 ± 0.60	14.45 ± 0.68
650	12.86 ± 0.44	12.17 ± 0.48	12.88 ± 0.52
720	14.01 ± 0.45	14.10 ± 0.45	10.81 ± 0.83
800	13.69 ± 0.16	13.44 ± 0.16	13.61 ± 0.22

Table 5: Fitted values of g_0^2 from three assumptions for HBE.

T(MeV)	OPE+HBE	Experiment	Discrepancy
142	4.829	4.63 ± 0.13	-0.20 ± 0.13
210	6.211	6.00 ± 0.07	-0.21 ± 0.07
325	7.445	7.36 ± 0.07	-0.08 ± 0.07
425	7.882	7.98 ± 0.11	0.10 ± 0.11
515	8.004	7.99 ± 0.13	-0.01 ± 0.13
650	7.913	8.05 ± 0.13	0.14 ± 0.13
800	7.635	8.17 ± 0.12	0.52 ± 0.12

Table 6: Comparison of fitted values of $\bar{\epsilon}_3$ with HBE. Units are degrees.

T(MeV)	OPE	HBE	OPE+HBE	Experiment	Discrepancy
142	1.205	-0.011	1.194		
210	1.905	-0.026	1.879	1.83 ± 0.07	-0.05 ± 0.07
325	2.881	-0.067	2.814	2.71 ± 0.05	-0.11 ± 0.05
425	3.555	-0.119	3.436	3.34 ± 0.08	-0.10 ± 0.08
515	4.060	-0.183	3.877	3.79 ± 0.09	-0.09 ± 0.09
650	4.681	-0.313	4.368	4.37 ± 0.10	0.00 ± 0.10
800	5.234	-0.509	4.725	4.86 ± 0.11	0.13 ± 0.11

Table 7: Comparison of fitted values of $\bar{\epsilon}_5$ with HBE. Units are degrees.

T(MeV)	OPE	HBE	OPE+HBE	Experiment	Discrepancy	Parameter
142			-1.676			3G_3
210			-2.860	-3.20 ± 0.16	-0.34 ± 0.16	
325			-4.647	-3.95 ± 0.13	0.70 ± 0.13	
450			-5.835	-5.26 ± 0.19	0.58 ± 0.19	
515			-6.593	-5.84 ± 0.17	0.75 ± 0.17	
650			-7.222	-6.96 ± 0.20	0.26 ± 0.20	
800			-7.346	-6.38 ± 0.16	0.97 ± 0.16	
800	-2.278	-0.091	-2.369	-2.20 ± 0.10	0.17 ± 0.10	3I_5
142	3.263	0.209	3.472			3G_4
210	4.931	0.427	5.358	5.84 ± 0.14	0.48 ± 0.14	
325	7.241	0.776	8.017	7.84 ± 0.20	-0.18 ± 0.20	
425	8.856	0.934	9.790	8.71 ± 0.15	-1.08 ± 0.15	
515	10.081	0.900	10.981	9.67 ± 0.16	-1.31 ± 0.16	
650	11.618	0.496	12.114	10.80 ± 0.20	-1.31 ± 0.20	
800	13.014	-0.433	12.581	10.20 ± 0.18	-2.38 ± 0.18	
800	5.199	0.300	5.499	5.25 ± 0.16	-0.25 ± 0.16	3I_6
142	-0.467	0.195	-0.272			3G_5
210	-0.825	0.454	-0.371			
325	-1.410	1.031	-0.379	-0.52 ± 0.20	-0.14 ± 0.20	
425	-1.872	1.609	-0.263	-0.70 ± 0.14	-0.44 ± 0.14	
515	-2.250	2.158	-0.092	-0.36 ± 0.17	-0.27 ± 0.17	
650	-2.756	2.999	0.243	0.11 ± 0.12	-0.13 ± 0.12	
800	-3.245	3.901	0.656	0.18 ± 0.19	-0.48 ± 0.19	
800	-1.219	0.676	-0.543	-0.69 ± 0.07	-0.15 ± 0.07	3I_7
800	-2.495	-0.095	-2.590	-3.39 ± 0.09	-0.80 ± 0.09	1H_5

Table 8: Comparison of fitted values of $I = 0$ G and H waves with HBE. Units are degrees.

T(MeV)	$\bar{\epsilon}_3$	3G_3	3G_4	3G_5	$\bar{\epsilon}_5$	3I_5	1H_5	3I_6	3I_7
210	✓	✓	?	✓	✓	✓	✓	✓	✓
325	✓	✗	✓	✓	✓	✓	C	✓	✓
425	✓	✗	✗	?	✓	✓	C	✓	✓
515	✓	✗	✗	✓	✓	C	C	C	✓
650	✓	?	✗	✓	✓	C	C	C	✓
800	✗	✗	✗	✗	✓	C	C	C	✓

Table 9: Summary of the measure of agreement between HBE and experiment for $I = 0$ phase shifts; ✓ indicates agreement, ✗ disagreement, L indicates that HBE is uncomfortably large, and C indicates that a correction has been applied to HBE predictions using experimental data from G waves.

T(MeV)	g_c^2
142	13.13 ± 1.70
210	11.84 ± 0.78
325	13.99 ± 0.45
425	12.68 ± 0.42
515	14.46 ± 0.43
650	14.23 ± 0.34
800	13.47 ± 0.31

Table 10: Fitted values of g_c^2 .

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